# Weighted Point-to-Line ICP for 2D Laser Scan Based SLAM Application

Lei Zhang○, 최성인, 박순용
경북대학교 전자전기컴퓨터학부, 경북대학교 전자전기컴퓨터학부, 경북대학교 IT 대학 컴퓨터학부
zhanglei@vision.knu.ac.kr, ellim5th@naver.com, sypark@knu.ac.kr

## Abstract

In this paper, the laser scan registration problem is solved using a weighted point-to-line metric. Consecutive laser scans are registered using the proposed algorithm to retrieve the robot motion. Since the point-to-line ICP is prone to poor initial alignment, a weighting scheme is adopted to reduce the impact of large rotation error. Experiments illustrate a great improvement comparing with the original ICP and point-to-line ICP.

## 1. Introduction

Simultaneous Localization and Mapping (SLAM) is the most fundamental research area in mobile robotics and has received considerable attention for the last 10 years. Many applications in robotics use techniques to estimate the relative displacement of a mobile robot. The iterative closest point (ICP) is one of the most popular algorithms to register consecutive laser scans which are retrieved from a 2D laser sensor installed on board.

After the ICP algorithm was originally proposed by Besl and McKay [1], many variants have been introduced based on the basic concept of ICP. Minguez et al. proposed a metric-based algorithm to estimate the robot displacement [2]. A new geometric distance measure was introduced by the authors, which both the translation and rotation are taken into account at the same time. Censi proposed an ICP variant that uses the point-to-line metric [3]. The point-to-line metric has the property of quadratic convergence and iterates in a finite number of steps. A exact closed-form for minimization of this kind of metric was also given by the author. Hong et al. proposed a method to correct the scan distortion by estimating the rangefinder's velocity [4]. In addition, an effective outlier rejection is introduced during the iteration of velocity update by comparing the estimated transformation from the previous step and the current sensing area. An ICP based polar point matching algorithm was proposed in [5]. The proposed method is based on the fact that the laser scan data natively uses the polar coordinate instead of the Cartesian coordinate. PPRM (Polar Point Matching Rule) was introduced for points in the polar coordinate along with the corresponding fast correspondence search algorithm.

Since ICP is a non-linear method, it can fall into a local minimum due to outliers, occlusions and poor initial alignment. In this paper, we propose a Weighted Point-to-Line (WPL) ICP algorithm. The point-to-line metric has the advantage of quadratic convergence, but is prone to poor initial alignment. A weighting scheme is utilized in this paper to reduce the impact of large rotation error and sensor noises. False correspondences are rejected based on a modification of the Fractional ICP [6], which greatly increases the robustness of the conventional point-to-line ICP framework. Experiments with respect to SLAM are conducted, are better results are yielded comparing with the original ICP and the Point-to-Line ICP algorithms. Besides, 3D feature map is built applying the rigid-body transformation computed from WPL-ICP.

This paper is organized as follows. Section 2 presents an overview of WPL-ICP. Section 3 gives an outline of the proposed algorithm. Experimental results are presented at Section 4.

## 2. Weighted Point-to-Line ICP

This section gives a full description of the proposed method including the ICP algorithm, rejection of false correspondences and the weighted point-to-line metric.

### 2.1. The ICP Algorithm

Given two laser scans, the *model set M* and the *data set P*, with an initial alignment, the ICP algorithm is based on an iterative process to find the correspondences between $P$ and $M$, and compute the rigid-body transformation to minimize the mean square errors. The error function of the point-to-point metric is defined as:

$$T=(R,t)=\arg\min_{(R,t)}\left\{\sum_{i=1}^{N_p}\left\|m_j-Rp_i-t\right\|^2\right\}, \tag{1}$$

Where $N_p$ is the number of point in $P$, $m_j \in M$ and $p_i \in P$. $m_j$ is the closest point of $p_i$, which is defined as the correspondence in the ICP algorithm. For each point in the *data set P*, search the closest point in the *model set M* to establish a correspondence. A naive search for the closest point has the time complexity O(n$^2$), which is very time-consuming. In order to accelerate the process, k-D tree is applied, so the time complexity will be reduced to

O(nlogn).

After the correspondences are established, the rigid-body transformation $T$ is computed as illustrated in Equation (1). A simple and robust approach to compute $T$ is the SVD-based Least Squares minimization [7]. Assume that for each point $p_i$ in $P$, **cp**($p_i$) is considered as the closest point operation on $p_i$ , and returns a point $m_j$ in $M$. The cross-covariance matrix of $p$ and $m$ is computed as the following equation:

$$H = \sum_{i=1}^{N_p} (p_i - \overline{P}) \cdot (cp(p_i) - \overline{M'}), \tag{2}$$

where

$$\overline{P} = \frac{1}{N_p} \sum_{i=1}^{N_p} p_i \text{ , and } \overline{M'} = \frac{1}{N_p} \sum_{i=1}^{N_p} cp(p_i). \tag{3}$$

The rotation matrix $R$ can be found by SVD of $H$, and the translation vector is computed as:

$$t = \overline{M'} - R\overline{P}. \tag{4}$$

## 2.2. Rejection of False Correspondences

Due to outliers, occlusions and sensor noise, the ICP algorithm can fall into a local minimum easily. Since the consecutive laser scans only overlap partially, the criterion of the closest point can result many false correspondences from the non-overlapping area. In this paper, we use the Fractional ICP algorithm to reject false correspondences. Only the correspondences which minimize the Fractional RMSD (Root Mean Square Distance) are used for Least Squares minimization:

$$\text{FRMSD} = \frac{1}{f^{\lambda}} \sqrt{\frac{1}{|P_f|} \sum_{p_i \in P_f} \| p_i - cp(p_i) \|^2}, \tag{5}$$

where $P_f$ represents the subset of $P$, $f \in [0,1]$ and $\lambda$ is set to 2.0.

After the correspondences are established, distances between corresponding pairs are sorted in ascending order. There are only $N_p$ fractions need to be considered as each prefix of the sorted order represents a fraction $f$. For each fraction, RMSD is updated to compute the FRMSD. If the smallest FRMSD is yielded at $i$th correspondence in the sorted order, the fraction $f$ is set to $i/N_p$ and $P_f$ represents the subset of $P$ that contains $i$ points with the smallest RMSD. The rest of the correspondences are rejected as outliers.

Assume the squared distance of the $i$th correspondence, which yields the smallest FRMSD, to be $SD_i$, the correspondences with squared distance larger than a threshold computed as the following equation are rejected:

$$t = \begin{cases} SD_i + median_{SD} & \text{, if } i > \frac{N_p}{2} \\ SD_i & \text{, otherwise} \end{cases}, \tag{6}$$

where $median_{SD}$ is the median value of the squared distance of all correspondences. Due to the sensor noise and the misalignment of initial position, it is expected to use relative more correspondences than the case of the smallest FRMSD is yielded. As $P$ and $M$ are aligned gradually, the median value decreases and more false correspondences are rejected. When the index of the correspondence in the sorted order is less than half of $N_p$, which is the index of median, P and M are considered to be “close enough”. Only $SD_i$ is used in this case.

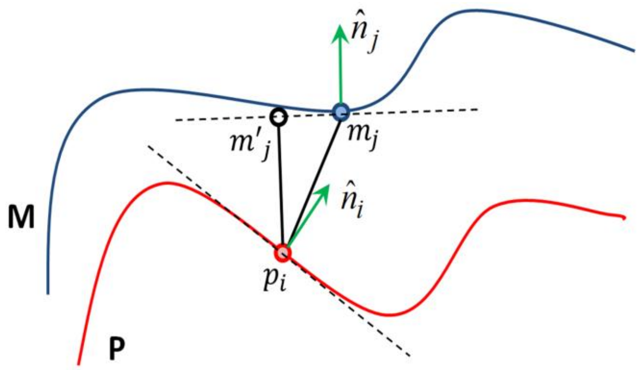



Figure 1: minimization of the Euclidean distance from $p_i$ to the tangent line through $m_j$.

## 2.3. Point-to-Line Metric

In order to speed up the convergence, the point-to-line metric is utilized in this paper. The point-to-line metric has quadratic convergence instead of linear convergence of point-to-point metric. Equation (1) is modified as the following equation:

$$T = (R,t) = \arg\min_{(R,t)} \left\{ \sum_{i=1}^{N_p} \hat{n}_j \| m_j - Rp_i - t \|^2 \right\}, \tag{7}$$

where $\hat{n}_j$ is the normal vector of tangent line going through point $m_j$. The point-to-line metric minimizes the Euclidean distance from $p_i$ to the tangent line through $m_j$, which greatly increase the robustness of the ICP algorithm against sensor noises.

As shown in Figure 1, the point $m_j$ is the closest point of the point $p_i$. The tangent line going through $m_j$ can be computed using additional two points from left and right sides. After the normal vector $\hat{n}_j$ is obtained, the point $m_j'$ on the tangent line going through $m_j$ can be computed using the following equations:

$$m_j' = p_i + w(D_{ij} \cdot \hat{n}_j), \tag{8}$$

where

$$D_{ij} = \frac{\hat{n}_j \cdot (m_j - p_i)}{|\hat{n}_j|}, \tag{9}$$

and

$$w = \cos\left(\frac{1}{2}\arccos\left(\frac{\hat{n}_j \cdot \hat{n}_i}{|\hat{n}_j| \cdot |\hat{n}_i|}\right)\right). \tag{10}$$

When $p_i$ and $m_j$ are facing the opposite directions, the probability that $p_i$ and $m_j$ is a false correspondence is very large. The $w$ is used as a weighting factor to reduce the impact of this kind of false correspondence and improve the robustness to outliers.

## 3. Outline of WPL-ICP Algorithm

The pseudo code of the WPL-ICP algorithm is proposed in Algorithm 1, and some details of implementations are discussed.

**Algorithm 1: WPL-ICP**

---

**Input :** two laser scans: the *data set* $P = \{p_i, i = 1, \cdots, N_p\}$, and the *model set* $M = \{m_j, j = 1, \cdots, N_m\}$.

**output:** the rigid-body transformation *T*, which is shown in Equation (7).

---

Subsample the *data set P* uniformly ;
IterNum = 0;
**while** ( IterNum < MaxIterNum **&&** *not converged* ) **do**
    IterNum = IterNum + 1;
    **for** ( every point $p_i$ in the sampled *P* ) **do**
        Find the closest point $m_j$ in *M* ;
    **end**
    Compute the threshold *t* in the Equation (6) ;
    Reject false correspondences with distance larger than *t*;
    **for** ( every true correspondence $p_i$ and $m_j$ )
        Compute the point $m_j'$ on the tangent line of $m_j$, using Equations (8), (9) and (10);
    **end**
    Compute the rigid-body transformation *T* to minimize the point-to-line metric, using SVD-based Least Squares minimization ;
    Apply *T* to the *data set P* ;
**end**

### 3.1. Implementation

The WPL-ICP algorithm is implemented using C++. In order to keep the simplicity, uniform sampling is preferred in this paper. The Approximate Nearest Neighbor (ANN) Library [8], which is originally programmed by Mount and Arya, is used to perform the closest point search as k-D tree. The algorithm terminates when the difference of Mean Square Error between two consecutive iterations is close to zero, or the number of iterations exceeds a predefined maximum.

## 4. Experimental Results

The laser scans used in experiments were obtained from a laser sensor. All the experiments were done off-line on a PC with Intel Core i5 2.66GHz CPU, 4GB main memory and Window 7 OS.

### 4.1. Pair-Wise Registration

We tested the performance of WPL-ICP comparing with other two algorithms: the original ICP (OICP) and the Point-to-Line ICP (PLICP). Uniform sampling and SVD-based Least Squares minimization are applied to all the three algorithms. In spite of outliers, sensor noise and poor initial alignment, as shown in Figure 2(a), WPL-ICP still converged to the global minimum while other two algorithms converged to a bad local minimum.

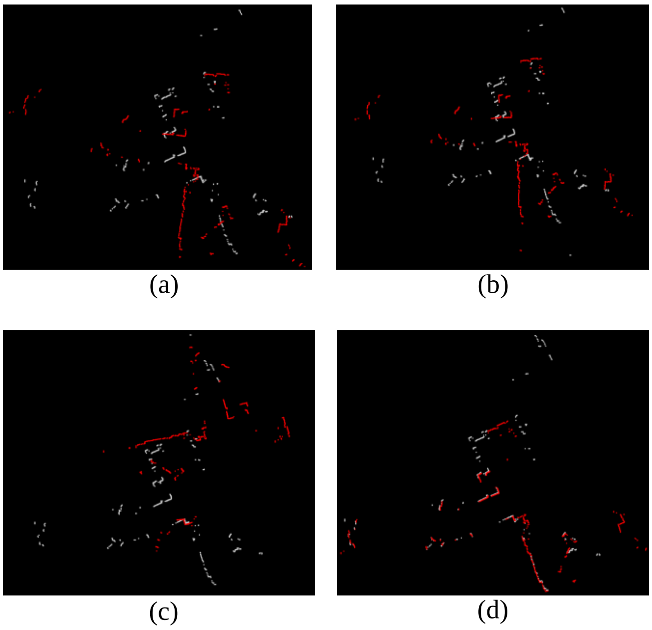



Figure 2: (a) Initial position of the *data set* (red) and the *model set* (white). (b) Registration result using OICP. (c) Registration result using PLICP. (d) Registration result using WPLICP.

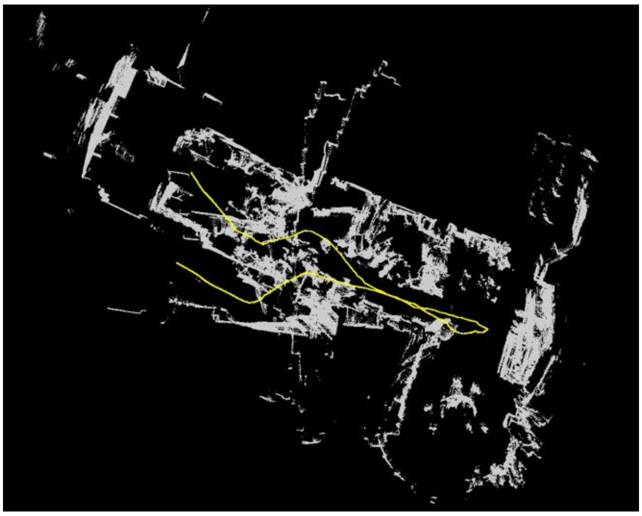
### 4.2. 2D SLAM

For 2D SLAM experiments, we use odometry information retrieved from the robot as the initial alignment. The experiments were conducted in an indoor environment. The robot traveled approximately 50 meters while 3780 laser scans were obtained. The field of view of the laser sensor is 270 degrees and the angular resolution is 0.25 degree. After uniform sampling, about 400 points were used for scan registration.

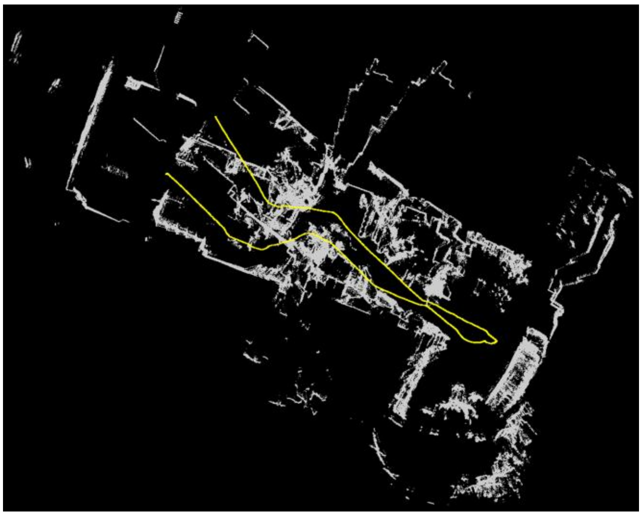
In the SLAM process, small error accumulated between consecutive laser scans in a long sequence. The quality of localization and mapping was degraded, which led to inconsistent scenes and trajectories, as shown in Figure 3(a) for the case of OICP and Figure 3(b) for the case of PLICP. The accumulated error was spread over the whole set for the case of WPLICP, which improved the quality of SLAM result.

### 4.3. 3D Feature Map Building

Along with the laser sensor, a stereovision camera was also installed on the mobile robot. Depth of the points from the real-world scenes were retrieved using stereo matching algorithm. Since dense stereo matching requires a lot of computation time, feature stereo matching was preferred in this paper to increase the efficiency. Features were extracted using the FAST feature detector. After stereo matching, 3D feature point sets were obtained. Applying the rigid-body transformation computed using WPL-ICP algorithm, the map containing 3D features was built, as shown in Figure 4(a).



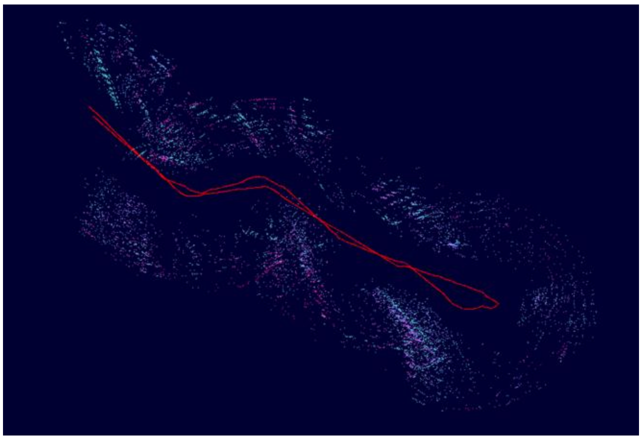
(a)



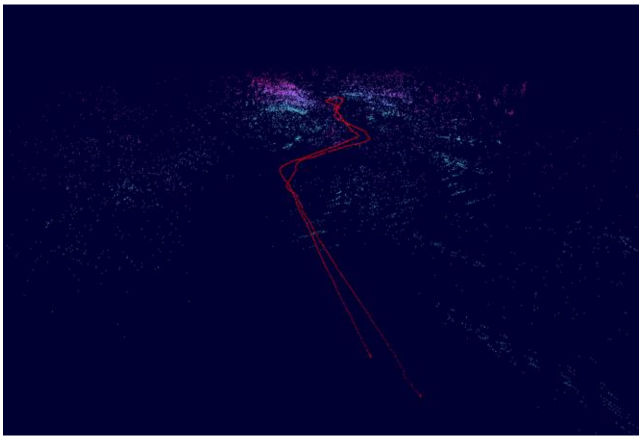
(b)

(c)

Figure 3: SLAM experimental results using (a) OICP (b) PLICP (c) WPLICP. White points are the built map and yellow points are the trajectory of the robot.



(a)



(b)

Figure 4: 3D feature map and the trajectory of the mobile robot. The red points on the map are from relative high position in the real-world scenes, while the blue points are from low position in the real-world scenes. (a) Top view. (b) Front view.

## 5. Conclusions

A variant of ICP algorithm, called Weighted Point-to Line ICP, is proposed in this paper to solve the 2D SLAM problem. Fractional ICP is adopted to reject false correspondences to reduce the impact of outliers. Instead of the point-to-point metric, a weighted point-to-line metric is applied to speed up the convergence and increase the robustness to sensor noises and poor initial alignment. The WPL-ICP is tested using laser scans grabbed by a laser sensor. The experimental results show a great improvement comparing with other algorithms. Future work includes adjustment and loop-closing for SLAM.

## 감사의 글

본 연구는 지식경제부 및 정보통신산업진흥원의 로봇 비전 연구의 한계 상황 돌파를 위한 핵심 기술 개발 지원산업, 및 국방과학연구소의 "소형무인로봇의 자율복귀를 위한 스테레오 비전 기반 위치인식 기술 개발"과제의 지원으로 수행되었음.

## Reference

[1] P. Besl and N. McKay, “A Method for Registration of 3-D Shapes,” IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 14, no. 2, pp. 239-256, 1992.
[2] J. Minguez, L. Montesano and F. Lamiraux, "Metric-Based Iterative Closest Point Scan Matching for Sensor Displacement Estimation," IEEE Trans. on Robotics, vol. 22, no. 5, pp. 1047-1054, 2006.
[3] A. Censi, “An ICP Variant using a Point-to-Line Metric," in Proc. of the International Conf. on Robotics and Automation, 2008.
[4] S. Hong, H. Ko, J. Kim, “VICP: Velocity Updating Iterative Closest Point Algorithm, ” in Proc. of the International Conf. on Robotics and Automation, 2010.
[5] R. Guo, F. Sun, J, Yuan, “ICP based on Polar Point Matching with application to Graph-SLAM,” in Proc. of the International Conf. on Mechatronics and Automation, 2009.
[6] J. Phillips, R. Liu and C. Tomasi, “Outlier Robust ICP for Minimizing Fractional RMSD,” in Proc. of the International Conf. on 3-D Digital Imaging and Modeling, 2007.
[7] K. S. Arun, T. S. Huang, and S. D. Blostein, “Least Square Fitting of Two 3-D Point Sets,” IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 9, no. 5, pp. 698-700, 1987.
[8] D. M. Mount and S. Arya. ANN: A Library for Approximate Nearest Neighbor Searching. [Online] Available: http://www.cs.umd.edu/～ mount/ANN/.